Open Channel Flow

14.1 Introduction to Open Channel Flows

14.1.1 Properties of Open Channel Flows

Definitions:
- **Open channel flow** (free surface flow or gravity flow): liquid flow that has an interface between its upper layer and a gas
- **Free surface**: interface between a liquid and a gas with constant pressure (almost always, atmospheric pressure)
- **Reach**: a continuous stretch of a waterway where properties may be simplified to be uniform

Examples of open channel flow:
- ocean waves
- river currents
- rainfall overland flow
- flows in canals and culverts

Characteristics of open channel flow:
- Most common flow phenomenon on the Earth’s surface
- Elevation of the free surface varies with velocity.
- Most natural open channel flows are 3-dimensional.
- Engineering practice typically simplifies these flows as 1- or 2-dimensional.
  - Examples of 1D flows: flows in rivers, channels, and culverts
  - Examples of 2D flows: meandering rivers in flat areas, patterns in lakes
- Most open channel flows in nature are turbulent.

1D flows are usually modeled assuming that the velocity across the entire cross section is equal to the average velocity of the section:

\[ \bar{v}_{avg} = \frac{\int_A \bar{v} \cdot d\bar{A}}{A} \]

While a one-dimensional approximation may seem too simplistic compared to other methods we have studied previously, in practicality, this approximation provides useful results and is used widely in the field of hydrologic and hydraulic engineering.

14.1.2 Classification of Open Channel Flows

Open channel flows are classified as either steady or unsteady, and uniform or nonuniform. We have discussed the meaning of these classifications previously. The velocity and depth of open channels can be a function of position or time in the following combinations:
<table>
<thead>
<tr>
<th>Classification</th>
<th>Average velocity</th>
<th>Depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>steady uniform flow</td>
<td>( v = \text{const} )</td>
<td>( y = \text{const} )</td>
</tr>
<tr>
<td>steady nonuniform</td>
<td>( v = f(x,y,z) )</td>
<td>( y = f(x,y,z) )</td>
</tr>
<tr>
<td>unsteady uniform</td>
<td>( v = f(t) )</td>
<td>( y = f(t) )</td>
</tr>
<tr>
<td>unsteady nonuniform</td>
<td>( v = f(x,y,z,t) )</td>
<td>( y = f(x,y,z,t) )</td>
</tr>
</tbody>
</table>

Steady uniform flow is a simplification that rarely, if ever, occurs in nature. However, we will start with this premise to derive equations for flow rate through a cross section.

Within steady nonuniform flow, the rate at which changes in velocity across cross sections occur may vary. In the case of gradually varied flow, the velocity and depth of flow change in a slow manner across an extensive reach of a channel. Examples include flow in a live stream during dry weather, and backwater from a dam. In rapidly varied flow, a rapid change in depth and velocity occurs. Examples include hydraulic jumps, and flow in the vicinity of an obstruction. If the depth of nonuniform flow decreases downstream, it is called accelerated nonuniform flow, conversely if the depth of a nonuniform flow increases downstream, it is called retarded nonuniform flow.

Unsteady uniform flow rarely, if ever, occurs in nature, and is usually not modeled by engineers.

Unsteady nonuniform flow is the most common form of open channel flow in nature, but also the most difficult one to model. Examples include flood waves in rivers and moving hydraulic jumps. These flows are usually approximated as steady nonuniform flows and are outside of the scope of this course. However, we will learn how to model some unsteady nonuniform phenomena using a quasi-steady model.

### 14.1.3 Overview of the Froude Number

Open channel flows are dominated by gravitational forces. The effect of these forces can be studied using the Froude number.

\[
Fr = \frac{v}{\sqrt{gL}}
\]

The Froude number represents the ratio of inertial forces to gravitational forces. In this dimensionless parameter, \( v \) is the average velocity of a cross section, and \( l \) is a length scale. If the inertial forces (represented by the velocity scale) are greater than the gravitational forces (to wit, if \( Fr > 1 \)), the flow will be characterized by relatively high velocities and shallow depths. This is called supercritical flow (rapid flow). If \( Fr < 1 \), the flow is characterized by low velocities and deep depths, and is called subcritical flow (tranquil flow). When the ratio of inertial and gravitational forces is 1, the flow is described as critical flow.

### 14.1.4 Hydrostatic Pressure Distribution

Most open channels can be presumed to have little to no vertical acceleration, and their streamlines can be assumed to be parallel. When this happens, the pressure distribution along the vertical direction is hydrostatic. Since \( z \) is the variable commonly used for an elevation, engineers typically use the variable \( y \) to refer to channel depth, and \( z \) will refer to the elevation of the channel bottom. Therefore, the absolute pressure at the bottom of a channel is calculated as:

\[
P_{\text{abs}} = P_{\text{atm}} + \gamma y
\]

while the gage pressure is:

\[
P = \gamma y
\]
14.2 Steady Uniform Flow

14.2.1 Channel Geometry

Definitions:

- **regular section**: cross section that does not vary along the length of the channel
- **irregular section**: cross section that varies along the length of the channel
- **prismatic channel**: channel with a regular cross section

In this course, channel cross sections will be simplified into four representative shapes: rectangular, triangular, trapezoidal, and circular. Trapezoidal cross sections are most commonly used to represent natural channels when a simplified model is necessary.

If we consider a rectangular channel with base $b$ and depth $y$, we can calculate the following geometric parameters (remember that when it comes to geometry, $P$ represents the wetted perimeter):

$$ A = by $$

$$ P = b + 2y $$

A trapezoidal channel requires mode dimensions in order to calculate its geometric parameters. Usually, the bottom with $b$ and depth $y$ are given. If the top width $B$ is also given, then the area is:

$$ A = \frac{y(b + B)}{2} $$

However, a more common way of expressing geometry is by giving the side slopes, $m_1$ and $m_2$ instead of the top width; in which case the geometric parameters are:

$$ A = by + \frac{1}{2}y^2(m_1 + m_2) $$

$$ P = b + y\left(\sqrt{1 + m_1^2} + \sqrt{1 + m_2^2}\right) $$
These equations can be used to find geometric properties for rectangular and triangular sections. Note that a rectangular section is a trapezoidal section with side slopes $m_1 = m_2 = 0$. A triangular section is a trapezoidal section with bottom width $b = 0$:

$$A = \frac{By}{2} = \frac{(m_1 + m_2)y^2}{2}$$

$$P = \sqrt{y^2 + (ym_1)^2} + \sqrt{y^2 + (ym_2)^2}$$

Circular cross sections represent flow in pipes. As the geometric properties of a partially-full circular pipe are complicated to calculate, most examples in this lesson will cover half-full circular pipes, for which case those properties are:

$$A = \frac{\pi}{8}D^2$$

$$P = \frac{\pi}{2}D$$

Up to this point we’ve treated full pipes as pressurized. However, a common method for estimating the capacity of a circular pipe is to consider the area and wetted perimeter of a full pipe. More details on this will be shown later. The geometric properties for a full pipe are:

$$A = \frac{\pi}{4}D^2$$

$$P = \pi D$$

The geometric properties of cross sections are necessary in order to determine the effects of friction from the channel on flow. Friction is related to the hydraulic radius (covered in Lesson 10.6.4), which we’ve already defined to be $R = A/P$.

A composite cross section is one that consists of several subsections of different geometries. Composite cross sections are found most commonly in cases of overbank flow (where the main channel overflows and part of the flow travels through the banks of the channel).

### 14.2.2 Chézy Equation

In the late 18th century, Antoine the Chézy investigated the relationship between the velocity of flow in a channel and the channel geometry. His analysis concluded in the following relationship: $v^2 \propto RS$ which is commonly written as:

$$v = C\sqrt{RS}$$

This equation is called the Chézy equation. To understand the coefficient $C$ (known as the Chézy coefficient), let’s compare this equation to the one found using the Darcy-Weisbach equation. To do this, we’ll apply the energy equation for steady uniform flow to the open channel shown in the figure.
In this example, the pressure and velocity across the control surfaces remains constant (due to the flow being uniform). The difference in elevation between the control surfaces can be quantified as $z_{\text{in}} - z_{\text{out}} = \Delta z = L \sin \theta$. Accounting for this, the energy equation takes the form:

$$L \sin \theta = h_f$$

Let’s define the longitudinal slope of the channel as $S = \tan \theta = \Delta y/\Delta x$. Furthermore, most channels in nature have small longitudinal slopes, for which we can say that $\sin \theta \approx \tan \theta \approx \theta$. This gives us: $\sin \theta \approx S$. Writing into the energy equation, we get: $LS = h_f$.

The friction loss can be expressed using the Darcy-Weisbach equation for noncircular conduits ($h_f = f \frac{L v^2}{4R^2 g}$).

Substituting back into the energy equation and solving for velocity gives: $v = \sqrt{\frac{8gRS}{fL}} = \sqrt{\frac{8gS}{f}}$. Now, comparing this expression with the Chézy equation, we can relate the Chézy coefficient to the friction factor by:

$$C = \sqrt{\frac{8g}{f}}$$

This allows us to deduce that the Chézy coefficient depends on both the flow properties and the channel roughness.

### 14.2.3 Manning Equation

Irish engineer Robert Manning empirically related the Chézy coefficient to the channel roughness and hydraulic radius by: $C = R^{1/6}/n$ where $n$ is a surface roughness value known as Manning’s roughness coefficient. Manning’s findings gave the Chézy equation the form of:

$$v = \frac{R^{2/3} S^{1/2}}{n}$$

This equation is known as the Chézy-Manning formula, the Gauckler-Manning formula, or simply the Manning equation. The SI units of the surface roughness coefficient can be found through $n = \frac{R^{1/6}}{C}$ and $C = \sqrt{\frac{8g}{f}}$ to be $\frac{s}{m^{1/3}}$.

Since values of Manning’s roughness coefficient were measured in metric units, a conversion to English units required a conversion factor $k$, factored into the equation as:

$$v = \frac{k}{n} R^{2/3} S^{1/2}$$

where $k = 1$ for metric units, and $k = 1.486$ for English units (this is because $1 \frac{m^{1/3}}{s} = 1.486 \frac{ft^{1/3}}{s}$). As this equation applies to steady uniform flow, then the flowrate can be found by $Q = vA$.
\[ Q = \frac{k}{n} AR^{2/3} S^{1/2} \]

The flow depth for a channel under uniform conditions is called normal depth.

Some representative values of the roughness coefficient \( n \) include:

<table>
<thead>
<tr>
<th>Channel type</th>
<th>Average ( n )-coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wrought iron</td>
<td>0.016</td>
</tr>
<tr>
<td>Corrugated metal</td>
<td>0.024</td>
</tr>
<tr>
<td>Glass</td>
<td>0.010</td>
</tr>
<tr>
<td>Concrete</td>
<td>0.013</td>
</tr>
<tr>
<td>Wood</td>
<td>0.017</td>
</tr>
<tr>
<td>Clay</td>
<td>0.016</td>
</tr>
<tr>
<td>Brick</td>
<td>0.015</td>
</tr>
<tr>
<td>Natural stream</td>
<td>0.030</td>
</tr>
<tr>
<td>High grass floodplain</td>
<td>0.035</td>
</tr>
</tbody>
</table>

These values were obtained from Chow, *Open Channel Hydraulics* and represent the average \( n \)-coefficients recommended for use in design.

**14.2.4 Best Hydraulic Cross Section**

Definitions:

- **Conveyance**: the flow capacity of a channel’s cross section.

Engineers who design open channels often start with a known flowrate (called the design flow). Given this flowrate, it is their job to select a channel geometry that minimizes the amount of material used for the channel while maximizing the conveyance of the channel. From the friction resistance equations studied previously, we see that flow capacity depends on the hydraulic radius of the section. Therefore, for a given channel shape, conveyance is maximized if the hydraulic radius is maximized. This implies that the cross-sectional area must be as large as possible relative to the wetted perimeter.

Let’s consider a rectangular cross section with base \( b \) and depth \( y \). To find the right combination of \( b \) and \( y \) that provides the most conveyance with the least friction resistance, let’s first express \( y \) as a function of area:

\[ y = \frac{A}{b} \]

Then, the wetted perimeter can be expressed as a function of area and base:

\[ P = b + 2 \frac{A}{b} \]

If the wetted perimeter is a function of area, we want to find a minimum for this perimeter. We can do this by setting \( \frac{dP}{db} = 0 \):

\[ \frac{dP}{db} = 1 - 2 \frac{A}{b^2} = 0 \]

\[ b^2 = 2A = 2by \]
Therefore, the best hydraulic cross section (or most efficient cross section) in a rectangular channel is one where the base is equal to twice the depth of the channel. We can repeat this process for other geometric shape. It should be noted that this only considers conveyance as a factor. Engineers must also take into account excavation, easements, topography, and other factors to determine the right cross-sectional shape to use.